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"Tower of Knowledge"
Showing Stages in Mediaeval Education

The child got his arithmetic from Boethius, his geometry from Euclid, and his astronomy from Ptolemy, who are shown here in the upper part of the tower.

[The illustration is from the "Margarita Philosophica," 1504.]

THE MATHEMATICS TEACHER

Vol. XXIII No. 1

Edited by William David Reeve

Creating Interest in Mathematics through Special Topics

By Lena B. Hansen
Enid, Oklahoma

For a number of years every student taking mathematics in our senior high school has been required to develop some special assignment in the form of a booklet or poster. To be up to date we are now calling these assignments "projects." At the close of the year the best of these are sent to our annual school exhibition. These projects have been the means of creating greater interest in mathematics without in any way lowering the standard for the regular work.

Last year was an especially good year in which to introduce topics early in our geometry classes, because of the many geometric designs which appeared in the dresses and scarfs worn by the girls and the sweaters of the boys.

After a few weeks of geometry we held a geometry "style show" in charge of a committee, every student, however, being expected to contribute something. It was really surprising how many geometric designs were found, not only in dresses, scarfs, and sweaters, but in shoes, handbags, compacts, shirts, hose, hats, and costume jewelry. This led to a discussion of decorative and practical uses of geometry, and so our list of topics grew from suggestions by the students and teachers.

After the list was well under way each student chose a topic for his project. Not more than three of four from one class were allowed

to choose the same subject. Later, a few asked to change as some new idea was suggested. In order to make several topics profitable for the whole class, all were asked to contribute something to as many projects as possible. Thus, in studying *Geometry in Textiles,* we set aside one week in the fall and one in early spring in which all students were to bring in samples of materials containing geometric designs. Some had friends or relatives clerking in stores to help them, so the result was a wealth of material containing many different designs—parallel lines, angles, triangles, rectangles, parallelograms, trapezoids, circles, sectors, segments of circles, and polygons in all sorts of combinations were found. The study of congruence and similarity was much more interesting when it was discovered that in a piece of material the groups must be congruent or similar to make the material attractive.

When the material was collected it was divided among students who had chosen this topic. They added more as they found it, and made booklets giving construction of the figures, exercises suggested by them, and samples of the fabric.

The same plan was followed with Geometry in *Tiling, Linoleums, and Rugs; Geometry in Home Furnishings,* and *Geometry in Advertising.* The last of these proved particularly interesting because it brought out more unusual constructions. Advertisements were found containing the five- and seven-pointed stars, and this made the study of regular polygons seem more worth while.

Some of the topics suggested and chosen were not of such a nature that all could find material to contribute, but a list of topics was posted and students were encouraged to bring material they happened to find which would help others.

Geometry in Art brought out a very attractive booklet by one of the girls. She showed that many artists block out their work geometrically. The triangular or pyramid form seemed quite popular with a number of artists. Copies of Murillo's "Children of the Shell," "Can't You Talk" by Holmes, and "Saved" by Landseer illustrated this very vividly, while the circular form often used was illustrated by Raphael's "Madonna of the Chair." "The Last Supper" by Leonardo de Vinci also showed the triangular form in the figure of Jesus, though the rectangular forms predominated. Vertical lines, it was shown, created the impression of dignity, horizontal lines, repose, and diagonals, action. Appropriate pictures illustrated these ideas.

Geometry in Court was the title given by one student to an article in which geometry appeared before a jury of curriculum makers to show why it should be studied in high school.

An old book on logic fell into the hands of a boy who became interested in discovering syllogisms in theorems. The outcome was *Geometry and Logic,* a profitable study of several theorems showing how logic is the basis of geometry.

Geometry now began to take on a new meaning in the eyes of the students. One day a boy said, "What would the world be without geometry?" Immediately another boy wanted *A World Without Geometry* as his topic and wrote a clever little story of a planet where Euclidean geometry did not exist.

Historical topics were suggested for those who particularly enjoyed history. Some students who could draw well wanted to make reproductions of the pictures of some of the mathematicians. These were framed as were also some fine quotations printed by students who liked to do lettering. We have had boys tell us they could never forget nor fail to be inspired by such quotations as "The laws of nature are the mathematical thoughts of God."

Quite a number of the more mathematically inclined students wanted to solve more exercises; so such topics as *Five Proofs of the Pythagorean Theorem* (that by Leonardo de Vinci being of especial interest), *Practical Exercises in Geometry, Exercises on the Trapezoid, A Study of Parallelograms, The Circle,* and the like were given.

Some very attractive posters were contributed showing samples of fabrics with the largest number or most striking geometric designs, and also a variety of figures used in advertisements. A few clever cartoons were made illustrating some of the favorite cautions of the teachers such as "Don't jump at conclusions" and "Proving a proposition by what is to be proved is like trying to lift yourself by your boot straps."

It was more difficult to find a variety of material for the upper classes. One of the upper class boys had heard it argued that history and English were the only subjects taught in high school which had any influence on character building. He wanted to write on *Mathematics and Character,* and made a very good argument in favor of mathematics.

The students purchased, one year, a demonstration slide rule and sixteen ten inch rules, so *The History of the Slide Rule and Calculating Machines* is always a favorite topic with the upper classes.

Other projects chosen by upper class students deal with all possible applications of trigonometry which they are able to find, special problems in algebra, an extended study of graphs, construction of various solids and applications of solid geometry, and also reviews of magazine articles which have a bearing on the study or use of mathematics. A number of articles were found, such as an interview of the Wright brothers which appeared in the *Saturday Evening Post* of July, 1928, wherein great stress was laid on the use of trigonometry by the Wright brothers in their experiments in the construction of airplanes.

Other topics we have used are: *A Short History of Geometry; The Origin of Signs and Symbols; Egyptian, Babylonian, and Hindu Mathematics; The Origin of the Hindu-Arabic Numerals; History of the Lives of Several Mathematicians; The Origin of Weights and Measures; The Abacus; Superstitions about Numbers; Geometry in Men's Clothing; Geometry in Astronomy; Geometry in Architecture; Geometry in Bridge Building; Geometry in Windows; Geometry in Furniture; Out-of-Door Problems in Geometry; Mathematics in Nature; Mathematics in Music and Musical Instruments; Mathematical Terms Derived from Latin; Mathematical Terms Found in the Bible; Original Magic Squares;* and *An Original Poem, Story, or Play.*

The work on these projects is not allowed to be a burden to the student nor to take a great deal of his time; nearly all students finish several weeks before school closes. The result of this special work is a deeper interest in mathematics and an appreciation of its applications. We never hear the question, "What is the good of algebra or geometry?" In fact some of the posters brought in by a few students who fail to consult the teachers bear such statements as "Geometry, the Foundation of Knowledge" and "Geometry, the Key to Success," which, while they make us smile, show that even these poor students are impressed.

We try to make the work each year a little different. Something usually happens which gives us the keynote for the year as it did last year in the unusually large number of geometric designs used in clothing and accessories.

We have not used this whole list of topics any one year, but give it and a description of some of the projects, hoping it may prove helpful to some of the younger teachers of mathematics in opening up a field of possibilities for making mathematics vital to the student by appealing to his interest in things which he meets daily.

Value and Logic in Elementary Mathematics

By Fletcher Durell
Belleplain, New Jersey

About fifteen years ago our National Government brought suit for the dissolution of the United States Steel Corporation on the ground that this organization was a combination in restraint of trade and therefore was operating in violation of the Sherman Anti-Trust Law. In the year 1919 the final decision in this case was made by the Supreme Court of the United States.

In this decision it was admitted that the Steel Corporation was a combination of several originally competing businesses and that it had used its power to manage or direct the nation's steel business in certain ways. Nevertheless the ruling of the Court was that the Trust should not be dissolved and the Government suit was dismissed. It is interesting to inquire into the reasons which the Court gave for such a decision.

"The public interest is of paramount regard" is a statement in the decision. Again, "the public interest will be served" by dismissing the suit. In other words the public interest is the most supreme of all laws. Hence we may possibly infer that in the opinion of the Court, the most important clause in the Constitution of the United States is that clause in its Preamble which declares that it is the object of the Constitution "to promote the general welfare."

In the case of the Steel Trust, the Supreme Court not only makes the sweeping decision noted above, but it also goes on to investigate, and, in an informal way, to tabulate the reasons why in its opinion the Steel Corporation is a good Trust and a useful servant of the public. Thus, the opinion states that by specializing the products in its different plants this corporation has greatly lessened production costs. It has stabilized trade. It has increased foreign commerce. It has not used its power to crush or even diminish competition. It has improved the manufacture of steel by the establishment of a research department.

By this line of detailed reasoning the Court, in effect, constructed a kind of "yard stick" by which to measure each merger in business and determine whether it is for the public good or the reverse. It

is interesting to note that, as an outcome of all this, the practice has grown up whereby the leaders in a proposed consolidation of several business first submit a prospectus or outline of their plans to the office of the Attorney-General of the United States that it may decide whether the proposed merger is for the public interest and will be permitted.

So great have been the benefits flowing from the decision discussed above that it is not too much to say that the unprecedented prosperity of our country during the past eight years has been more due to this method of treating mergers than to any other one cause, and more, perhaps, than to all other causes combined.

The above illustration of the principle of value or function as dominating and overriding technical legality is somewhat exceptional in its dramatic qualities, yet if we had the space to make a survey of the history of law, we should find that, in the long run, value or utility considered in its broadest and fullest sense has always dominated legal forms; that statutory enactments and imperial decrees, in their fundamental nature have been local crystallizations in the stream of developing public welfare and interests, and that we can predict that the same will be even more definitely true in the future.

Thus, it is interesting to note that a new body of law is even now being worked out with respect to aviation as a new department of human activity, and that practically all decisions concerning this are being made not on any basis of abstract theories of human rights and vested interests, but on considerations of what is best for the public welfare.

The recent explicit and accelerated development of this principle on the part of the Supreme Court has been due, perhaps, more to Justice Oliver Wendell Holmes as a member of this Court, than to any other one man. In the year 1880, Judge Holmes, then serving on the Massachusetts bench, gave the Lowell Lectures in Boston. These lectures were published two years later as a book, entitled "The Common Law," a book "known to every jurist in the world as establishing the functional and relative view of the law now accepted as replacing the anatomical and morphological one." It is worthy of note that Judge Holmes was "a great friend and contemporary" of Professor William James, the apostle of pragmatism, a system of philosophy which makes value the supreme category in every de-

partment of life. While pragmatism as a general system of philosophy is somewhat cramped by certain mediæval survivals which cluster about it and while at times it seems to be seeking to establish a new authority and tradition of its own, yet at heart it contains a vital truth.

Hence it is interesting and should prove profitable to inquire what meaning this principle of the dominance of value and function, and the secondary place of formal logic, has for us in the study and teaching of mathematics.

Let us first make an application of this principle in the field of geometry.

Value and Logic in Geometry

The different courses in geometry or the various ways of studying the subject may be roughly classified in three groups, viz: (1) informal, (2) semi-formal, (3) formal.

Described with reference to the relative prominence in them of the two elements, value and logic, they may be termed the *value*, the *value-logic*, and the *logic* courses.

The first of these is what is commonly known as intuitive or informal geometry, and is now generally given as a part of the Junior High School curriculum.

The third or last of the three courses is the highly finished, logical treatment which is especially adapted for study as an elective by college students who are specializing in mathematics.

The second of the above courses, or that intermediate between the other two, is one in which the value principle is plainly dominant, with logical demonstration and theory secondary and used locally in an auxiliary way. It is this second method which we are mainly concerned with here and which we shall try to show is the most desirable in the senior high school.

Let us first consider some of the ways in which the course in geometry as now usually taught in the senior high school differs from the value-logic or semi-formal method which we advocate. The discussion of these points will include also some statement of the changes which would be necessary in order to make the idea of value relatively more prominent.

In studying high school geometry as this subject is ordinarily presented in our textbooks, pupils often complain that the first few theorems are obvious without any proof and that these proofs

at best seem like useless lumber. Often, moreover, such proofs contain elements of logical difficulty, require considerable mental effort for their mastery and as courses of reasoning are not very convincing. As a result, at the very beginning of the study, in the minds of many pupils, a violent prejudice against the whole subject is formed.

Pupils do not realize that the purpose of these first few demonstrations is not the discovery of new, unexepected, and valuable truths, but rather the simplification and systematic co-ordination of a number of rather obvious geometric facts. Even if this purpose were pointed out to pupils, probably few of them would really understand what was meant by such a statement; and, if they did understand it, would not regard the aim as of sufficient importance to justify the expenditure of so much mental effort. The values of such a simplification and co-ordination are to them so vague and remote as not to seem worth while, and there is much in their view which is essentially sound.

Teachers of geometry without number have remarked, that as to the textbooks theorems and demonstrations the first thrill of enthusiasm has really stirred and excited the class when the theorem that the sum of the angles of a triangle is two right angles has been proved. Here is something unexpected, surprising, and intriguing; with it are first realized the charm and value of a geometric demonstration.

Hence it has been proposed that in the senior high school course in geometry (naturally including the course for pupils preparing for college), the only formally demonstrated principles shall be those theorems which furnish something unexpected and plainly worth while and which give a real thrill. In particular all the geometric facts and properties up to the "angle-sum of a triangle" proposition shall be made clear and familiar and their usefulness evident by informal methods of different kinds including some that are experimental and mensurational.

In this course no special effort is to be made to limit to a minimum the number of first principles, assumed or treated informally. There may be as many of them as will add to the efficiency of the value-logic method of developing the subject. The values of these concepts and principles will be emphasized in every possible way, both as to inherent meaning and practical concrete application. The later formal

demonstrations will be local aids in the stream of developing values. No attempt will be made, or at least kept primarily in view, to build up a comprehensive formal logical theory complete in every detail. What has been said applies to solid as well as to plane geometry.

Professor Oswald Veblen is possibly the leading expert in this country on the logic of geometry. The writer once heard him read an elaborate paper on the first principles of a final and finished logical theory of the subject where a minimum of primal truths was to be used. In the discussion which followed it was interesting to note that Professor Veblen vigorously advocated the use in the ordinary high school, not of the highly technical theory which he had described but of a proximate value-logic method similar to, if not essentially the same as that which has been outlined in this paper, and that he urged textbook writers to produce a textbook embodying such a course. So long as college entrance requirements are what they are, the publication of such a book would, of course, be a costly experiment.

In recent years, however, the College Board and the New York Regents have taken notable steps toward modification of the old logical course in the direction here advocated, so that the present prescribed course is in a sense a compromise between the last two of our three courses. The present course, however, has this redeeming feature. It is possible for a teacher to take a textbook written on the present basis, and teach it in a value-logic way. Thus at first the early theorems may be treated and established in an informal way, and later near the end of the year, be studied in the usual formal demonstrative manner. Many teachers are today following this plan.

One or two other observations will be useful in this connection as showing both the desirability and possibility of further changes. Thus if we go back further historically we find that Euclid's treatment of geometry was written, as far as we can tell, primarily for adults or for mature students in what we may call the then University of Alexandria. Till a recent date (that is, less than 100 years ago) in our own country, the subject was taught only in colleges. In time, as new subjects were added to the college course of study and the secondary school system developed, the study of geometry was gradually pushed down into the academy and high school. But when thus transferred it was not modified to meet the new situation, and espe-

cially the more immature type of pupil. It was kept in its original abstract and highly technical logical form.

Moreover, Euclid had in mind to build up a pure logical system of geometry on an apparently minimal foundation. But this presentation of such a set of fundamental concepts and principles has not been accepted as entirely successful. Many attempts have been made to improve on Euclid's system of first principles, but no general agreement exists on the matter. The problem of finding a pure, *a priori* logical theory of geometry has not been solved in a generally accepted way. It is a question whether such a solution is possible. For instance it is doubtful whether a definition of so elementary an idea as a straight line, or a circle, or any adequate conception of such objects could ever be arrived at by purely ideal methods, and without any preliminary concrete experience with the same. If this is the case a pure logical theory going to the utmost roots of the subject seems impossible. Hence in all cases we shall merely have a question as to the degree to which useful concrete experience shall precede and be mixed in with abstract studies. This would leave us free in the high school curriculum to select that point on the scale best suited to secondary school psychology and other conditions.

It gives a broader view of the subject under discussion to note that changes like those proposed here have taken place in other subjects which have been transferred from the college to the high school. Thus, for example, when a course in economics was first introduced into the secondary school curriculum, it was essentially the old college course in political economy with its abstract laws and deductions. Lately this college element has been greatly modified and diminished. In place of the parts omitted, sections on personal, social, and business efficiency have been substituted, and in fact these new parts have become dominant. It is to be noted especially that these changes in economics in the line of putting the value element in control have been much greater than in the case of geometry. It should also be observed that college courses in economics are being modified somewhat in line with the practice of secondary schools as described above.

Finally in this connection mention should be made of the *Survey Course in Mathematics* by Professor N. J. Lennes for the Freshman class in college. This is essentially a value-logic course in Trigonometry, Analytical Geometry, and the Differential and Integral Calculus. Only so much of the theory of these subjects is given as is a

necessary preliminary to certain important and worth while applications of each of them, and much of this theory is treated in an informal way. In the words of the preface, "there is a considerable direct appeal to intuition where the severely logical path is not easily trod by the student." Hence in the college itself there seems to be some tendency in the direction which we regard as so desirable for the high school.

Algebra and Arithmetic

Much that has been said concerning the teaching of geometry applies also to algebra. Hence the statement of our position concerning this branch can be made more brief. As a representative topic in this connection we may take the axioms or fundamental laws often elaborately formulated at the outset and used in the solutions of equations. One of them, for instance, is the statement that if equals be added to equals the results are equal; another is that if equals be subtracted from equals, the results are equal; and so on. Hence they are very similar to some of the axioms of geometry, and what we shall say concerning them applies also to the latter in certain respects.

Concerning the use of the subtraction axiom in the representative algebraic example which he gives, Barber says (*Junior High School Mathematics,* page 86), "The word axiom . . . . introduced at this point [first solution of equations] does not seem to serve any useful purpose. . . . . In general the simple notion that if we take 50 from each side, the two sides will be equal, is easy enough to need no illustration. Doubtless we have sometimes confused the child by too much explanation of a very simple step."

As a matter of fact, these so-called axioms or laws are a part of the ordinary, everyday furniture of our minds, or of our unconscious mental apparatus. We use them many times each day in an implicit, informal way.

Take the following problem:

A rod 28 inches long is to be divided into two parts such that one of them shall be 6 inches shorter than the other. Find the length of the shorter part.

Before he has studied algebra the pupil might solve this problem by subtracting six from 28 and dividing by 2. That is, he subtracts 6 from the sum of the two lengths and also from 28 and divides each of the results by two. He thus uses the subtraction and division

axioms in an implicit way without any thought that his mental process could be formulated as these two laws.

If this problem is used as an introduction to algebra, the required quantities are symbolized as x and $x + 6$, and the solution is expressed in the equation form as follows

$$x + x + 6 = 28$$
$$2x = 22$$
$$x = 11$$

There is no more need of any formal use of the so-called two axioms in this algebraic solution than in the former arithmetical one. Hence it is possible to teach first year algebra in such a way that there is no formal use of these laws until after a full semester of study. In fact it is doubtful whether their explicit formulation ever adds anything to the pupil's sense of certitude or validity as to the soundness of the steps used in solving an equation; or to his assurance of the reliability of the result; or to his operational efficiency in working with equations. If described as aids in solving equations and problems they have a use in showing the unit or elemental sources of the efficiency powers of the equation. Too often, as ordinarily used or introduced they appear to the pupil as mere useless machinery whose purpose or function he cannot discover.

The wise thing, then, to do in beginning algebra is to introduce the child as soon as possible to the equation as a useful instrument, and much later and gradually in the stream of developing meanings and values to bring in, one by one, the parts of the logical technique as means of improving and developing the useful powers of this new implement of thought and investigation and of safeguarding its processes from error.

Similar remarks might be made concerning other special concepts and processes in algebra, such as identities and extraneous roots; properties of exponents like $x^0 = 1$, $x^{-n} = 1/x^n$; of radicals like $\sqrt{a^2b} = a\sqrt{b}$, $a/\sqrt{b} = a\sqrt{b}/b$, and so on. The meaning and useful properties of such ideas and principles are to be illustrated and made evident, before any effort is made to demonstrate them. In fact, in secondary mathematics any attempt at strict formal demonstration of such principles is probably out of place. Such principles should be simply "rationalized" or justified by some semi-concrete or special method.

In the teaching of arithmetic, the process of giving value the first place and of using formal logical deduction only locally and in ways that are easily understood has already gone far and is even now in control. For example, formal abstract proof of the rule for dividing by a fraction, viz.: "invert the divisor and multiply," is now rarely, if ever given; instead, the process is justified in some special way, or, as we say, rationalized. It is simply shown to be a reliable, useful, and efficient process.

Of like character in arithmetic is the emphasis on what is called the informational side of some topics. By this method of treatment, when teaching compound interest as practiced by savings banks, most if not all of the technical details of actual bank practice are omitted (or left as extra credit work for bright pupils) and the value of the services of such banks brought out more fully. The same general method applies to the teaching of stocks, bonds, insurance, and certain other topics.*

Methods of Application

In the application of the methods of instruction advocated in the preceding pages, certain principles of method, or even a distinct technique or science of it needs to be worked out in order to get the maximum of beneficial results and to avoid certain pitfalls. There are many different kinds of value, and many different ways in which each of these can be made evident and full of appeal. Also there are many diverse routes by which logical technique or methods may be made useful aids in obtaining values. Practically every mathematical concept, principle, or process has some special kind of item or value-logic technique connected with it.

An exhaustive consideration of all these various methods is beyond the scope of the present discussion. In the preceding pages a number of widely different representative illustrations of these various cases have been given, which will serve to give a general idea of the whole field.

However, two or three general considerations should be mentioned and briefly discussed in this connection.

In the first place it is important not to lose sight of the many and great values of logical theory and formal demonstration. Some of these have been mentioned incidentally in the preceding pages but

* For further treatment of this matter see the important article by Dr. Charles H. Judd in THE MATHEMATICS TEACHER, April, 1929.

it will be an advantage to state them more explicitly and fully at this point.

A general closely knit logical theory gives a comprehensive, unified grasp of a large number of principles, and facility in their choice and application. The tendency of the value principle is to become personal and individual, to scatter and even be chaotic. Hence logical theory supplies a needed supplementary unifying and organizing service.

Likewise logical demonstration not only often safeguards from error and gives a sense of absolute certitude and confidence, but also frequently arrives at a new principle with relatively little effort. In many cases this result could not have been obtained otherwise and is therefore of transcendent value.

On the other hand it is to be remembered that in some instances these logical values are too remote and indirect to be appreciated by a somewhat immature pupil. In other cases the logical processes required are too abstruse, labored, and technical to be grasped and mastered by the ordinary pupil.

In such cases, as has been indicated in various places, it may be well to omit, or mention as postponed for the time being, certain parts of a logical development, and to give theory only so far as it can be appreciated by the pupil.

On the other hand, the great value of logical theory and demonstration requires that we lay down and observe the complementary law or rule viz.: that as much logical theory shall be taught as possible and as early as possible. Just as in studying geography, the child should, as soon as he can, form a conception of the earth as a whole, so here, in a mathematical subject, its body of principles should be grasped as a whole as soon as the pupil is capable of so doing.

This line of thought also leads to another important principle of method, viz.: that as we proceed in school mathematics through arithmetic, algebra, and geometry, there should be a progressive increase in the amount and abstractness of logical theory and formal demonstration, or, in general, of logical technique.

In the next place, mention should be made of the teacher's part in these matters. This is larger and more important than in the old formal methods of instruction. In determining the kinds and relative amounts of logic and value to be used in so many diverse situations, the textbook can do much, but the teacher should also play an

important part in the form of adaptations, improvisations, and experiment. This is especially true if some method of individual instruction or of ability grouping of pupils is used.

One other item of technique in the efficient use of value and logic together, is of such importance that it also should receive special mention. This is the desirability of a somewhat regular alternation of the logic and value principles in their joint treatment. Such an alternation not only tends to give a valuable variety to the work but also provides a recurrent contrast which emphasizes each of the two factors and its values and aids in fixing them in the mind.

Advantages

In conclusion it will be useful to make a brief survey of the advantages of the methods of study and instruction which have been advocated in this paper. Some of these advantages have already been indicated in the course of the preceding discussion or are so obvious as to need only a brief mention. Others call for a fuller statement.

By the method proposed, pupils should find a more immediate and continuous interest in the work. Discouragement is avoided and joy is made possible. This helps solve the problem of motivation.

This should lead to greater and more spontaneous self-activity on the part of pupils, and hence to a better class-room democracy of teacher, pupil, and textbook. This as well as the cooperative way in which the various items of subject-matter are combined and made to work together in the value-logic textbook should engender the democratic habit of mind in the pupil. On the other hand, the old formal, mainly logical method of presentation and study, if successful was likely to instil a bureaucratic, or even imperialistic mental attitude. The more successful such study was, the more pronounced such an outcome was likely to be.

The value-logic method of study also supplies valuable elements of change and variety. With it the work may even become a succession of surprises and adventures, an uncovery of one new gift after another.

The method also makes it easier for the pupil to gain a real mastering of logical theory and progressive inference. Some pupils will be enabled to get this mastery who otherwise might fail to do so. A characteristic part of the method is to divide logical developments into

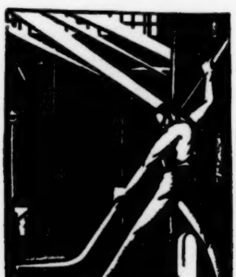
small units or parts, each of them definite and clear-cut, each having an evident value, and each capable of relatively easy assimilation. Hence, as just stated, it should lead to a better mastery of logical theory and by more pupils. In fact, such mastery may become a pleasure instead of a bugbear. Hence it opens the way, both positively and negatively, to a better and wider appreciation of mathematics and its values.

The value-logic method of study also leads to a better development of cooperative mathematics, as for instance, by the use of the large number and great variety of illustration of the principles of value which can be taken from other branches of mathematics when any one branch is being studied.

Broadest of all, the method of giving the principles of function and value the leading rôle and making formal technique secondary and auxiliary is one which should be used in many other, and perhaps all branches of study, and in life in general in all its details and relations. Hence in studying mathematics in the way here advocated the pupil is forming habits which have useful applications throughout the whole field of human thought and endeavor.

In other words it helps toward the acquisition of a general philosophy of life which if followed out will prove to be pragmatism stripped of its mediæval clogs and implications, a philosophy which in another connection we have termed *efficientism.*

At the end of our introductory illustration concerning a certain decision by the Supreme Court, it was pointed out that the recent extraordinary prosperity of the United States dates from the time when the value principle was given the leading place when dealing with corporations. Perhaps this story is a parable to show us the prosperity that may come to and through mathematics if it adopts a like policy in dealing with its own subject-matter.



A Fallacy in Geometry Reasoning

By H. C. Christofferson
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Oxford, Ohio

In The Mathematics Teacher for December, 1928, I attempted to expose a weakness in the *usual* proofs of beginning geometry and advocated a new and slightly different beginning which at the same time has the virtue of being more simple. The usual order of proofs is (1) congruence by side-angle-side; (2) congruence by angle-side-angle; (3) the theorem that the angles opposite the equal sides of an isosceles triangle are equal; (4) congruence by three sides; and (5) sometime later, the construction of an angle bisector and of an angle equal to a given angle. We are concerned here with only (3), (4), and (5).

It was a real joy to have appear in the current October issue of The Mathematics Teacher an article by a well known teacher and geometrician, Joseph A. Nyberg, showing what seemed to him to be an error in my logic. There are two reasons why this was such a joy to me. First it showed that someone had read my article. Second it gave me a chance to defend my contention. This I shall do and in in turn expose an error in Nyberg's pet proof. In fact his proof is a wonderful example of an unforgiveable error in logic, "petitio principii," reasoning in a circle.

Before exhibiting Nyberg's gross error it seems necessary to correct a false impression drawn from my defence of "a new beginning for geometry" and to repeat a portion of that defense.

The *usual* proof for the isosceles triangle theorem (3) is *to bisect the vertex angle* and then to prove the triangles thus formed congruent by side-angle-side. Mr. Nyberg contends that I claimed that this was the *only* proof for this theorem. No doubt I had not made myself clear, for neither "logician" nor "classroom teacher" would care to contend that any proof was the only proof for a theorem that can be proved in many different ways. My favorite proof is one that differs widely from the usual one and also from Mr. Nyberg's faulty one. At the conclusion of this response I shall give still

another proof that is very old but rigorously done and in the same sequence as that stated above.

Congruence by three sides (4) is next proved by using the isosceles triangle theorem and congruence by side-angle-side. Then several pages later in most textbooks, the construction of an angle bisector (5) is proved by reference to congruence by three sides. Mr. Nyberg in his recent and well written book has these three theorems on pages 40, 44, and 47 respectively.

The error in reasoning is evident unless skillfully avoided by a hypothetical construction. In the isosceles triangle theorem we use angle bisection which has not been previously proved, and in fact depends for its proof upon congruence by three sides which in turn depends upon the isosceles triangle theorem. Here is the circle reasoning.

Many teachers and some textbook writers are blissfully unaware of the danger in the reasoning pattern which they are boastfully exhibiting. The reasoning is made rigorous by the postulation of angle bisection or at least the postulation of the existence of such a bisector. It should take some such form as this, "Let us assume that line *CD* is the bisector of angle *C* since every angle has a bisector. If *CD* is the bisector, then the two triangles formed are congruent by side-angle-side. . . . ." It is evident that with this sequence in order to make the proof rigorous something must be postulated. The traditional way that is so often unnoticed and is so subtle has been to use this hypothetical construction, but the postulation of congruence by three sides is just as defensible from the standpoint of logic. Furthermore such postulation leads to a far more direct and simple beginning for geometry. This was my defense for a new beginning for Geometry. It has been attacked on the right flank by Mr. Nyberg, who contends that the proof outlined above is not the only proof for this isosceles triangle theorem. He then submits a proof for this theorem which he contends ought to be suitable for a "logician" since it has served him so well as a "classroom teacher." Let me now point out the error in Mr. Nyberg's proof, and then submit a proof without error which fits into the old sequence, but which is still inferior to the proof in the new sequence depending upon the postulation of congruence by three sides.

The error is one of circle reasoning almost exactly like that in the proof that has just been reviewed. It therefore provides further

defense for a new beginning for geometry rather than evidence against such a procedure. The proof given starts out like this:

"Given: Triangle ABC with $AC=BC$. Prove: Angle A=angle B.

(Figures omitted because unnecessary here now.)

(1) Construct triangle $A'B'C'$, making angle C'=angle C, $A'C'=AC$ and $B'C'=BC$."

The faulty reasoning would at once be apparent to any alert high school class. How do you construct angle C'=angle C? The construction is familiar to all teachers and its proof must depend upon congruence by three sides which in turn depends upon this theorem for its proof. The circle reasoning here is crudely apparent. No attempt even is made at a hypothetical construction of the angle.

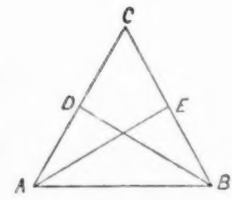
The rest of the proof is now of course useless. It is therefore needless to point that it depends upon superposition and is consequently based upon a technique whose rigor has been questioned for years.

Credit must be given Mr. Nyberg for objecting to his own proof in the last part of his article although the objection was on other grounds than logic. Here he was doing some good reasoning which was not in a circle except that it did condemn a proof which had just been presented with more or less conviction as to its usefulness. I am sure, however, that Mr. Nyberg is fully in agreement with the main issue of my article published last December, and merely took this means of displaying a clever though weak proof for the theorem criticised.

An old proof for this isosceles triangle theorem, while not simple, has at least the virtue of being rigorous. It is therefore submitted here to show that the *usual* form which I criticised is not the *only* form. This is still not the best proof by far, but only one of several of the same nature. What in my opinion is the best proof from the standpoint of logic and pedagogy depends upon a slightly different sequence whose exposition would be too long for this brief article.

Hypothesis: Triangle ABC with $AC=BC$.

Conclusion: Angle A equals angle B.



Proof:

(1) On AC and BC lay off $CD=CE$. Draw lines AE and BD.

(2) Triangle ACE is congruent to triangle BCD by side-angle-side.

(3) Therefore $AE=BD$ and angle $CDB=$angle CEA being corresponding parts of congruent triangles. (C. p. c. t. e.)

(4) $AD=BE$ and angle $ADB=$angle BEA—equals subtracted from equals.

(5) Therefore triangle ADB is congruent to triangle BEA by side-angle-side.

(6) Therefore angle $A=$angle B being corresponding parts of congruent triangles. (C. p. c. t. e.)

At most this proof depends only upon congruence by side-angle-side since even the construction of a line segment equal to a given line segment in the finest analysis depends at most upon this theorem.

Here is a proof that should appeal to a "classroom teacher" as well as to any "logician" both of whom are interested in exhibiting an errorless pattern of thought to the pupil. However, I do not wish to checkmate myself in this rebuttal. The new beginning for geometry which I advocated has decided preferences over even this logically rigorous proof for this seemingly crucial theorem which has been historically known as the "pons asinorum" meaning "the bridge over which fools could not hope to pass." At a later time I shall describe this new sequence and its unusual possibilities for a rigorous, simple, forceful new beginning for this remarkable subject of geometry that has been such a challenge to human thought and ingenuity for over 4,000 years. I cannot close this respone to Mr. Nyberg without disclaiming much of the honor which he bestowed upon me by calling me a "logician," and claiming to be, not so much of a logician as a classroom teacher who is interested in the pupil.

National Council Members

It is earnestly hoped that every member of the Council will attend the annual meeting at Atlantic City on February 21 and 22. See pages 63 and 64 of this issue for the program.

It is also the duty of every member to mark and return the official ballot on page 62.

Home Work Papers in First Year Algebra

By Nelson A. Jackson
Mount Hermon Boys' School
Mount Hermon, Massachusetts

Is it worth while in beginning algebra to look over carefully, mark, and return daily home work papers?

In order to obtain some idea of the value of home work papers in marking and also their value as a means of determining the pupil's grasp of the problems and exercises assigned, a colleague and I carried on the following study.

During a period of three weeks ten quizzes were given. There were four questions on each quiz, which were taken from the exercises assigned for home work. They were never the hardest exercises, but those of medium difficulty and often the easiest and shortest ones. The time for each quiz averaged from ten to fifteen minutes. The quizzes were carefully graded, and a record made of the number of exercises which were wrong and also the per cent value of the quiz as a whole. The daily home work papers were scored, and a record made of the number wrong of the four exercises which had been used on the quiz, also the per cent value of the paper as a whole. There were 613 quizzes examined, and the same number of home work papers. In all 2,452 exercises were compared.

The number of exercises which were wrong on the quizzes was correlated[1] with the number of the same exercises which were wrong on the daily home work papers. The coefficient of correlation was found to be .32. Then the per cent marks of the quizzes were correlated with the marks of the home work papers. In this case the coefficient of correlation was .49.

This study was not carried on until we had had our classes for about ten weeks, and pupil and teacher were well acquainted, so that the elements of nervousness and strangeness were eliminated.

Out of the 613 quizzes, 167 were failures; that is, had 2, 3, or 4 exercises wrong, while the home work papers of the same pupils had all the exercises correct, or only one wrong.

[1] The Pearson product-moment formula was used.

This study gives no one the privilege of drawing general conclusions or making definite deductions. It merely shows a tendency. It would seem to indicate that in first year algebra home work papers are of little or no value in determining the mark of the pupil; also that the average daily paper is of small worth to the instructor in judging how thoroughly the pupil has grasped the true meaning of problems and exercises assigned. It would seem that the principal value of the home work papers is found in the drill thereby furnished to the pupils. This is sufficient reason for requiring the papers.

Quizzes should be carefully scored, analyzed, and returned. Weak points should be discussed, and further drill should be given on them. It seems as if the true mark of the pupil is best obtained by taking the average of frequent quizzes, tests, and the final examination.

One interesting result of this study was its influence on the pupils. They seemed to grasp the idea that it was folly to depend on some one else for their daily work.

Later in the course after the above results had been discussed with the class another comparison of daily work and quizzes was made. Over a period of three weeks the home work was again carefully marked; during the same time two full period tests and one twenty-minute quiz were given. The median for the averages of the daily work was 62,[2] while that for the quizzes was 72. The median for the final marks for the course for the same pupils was 68. The second study, to my mind, simply confirms the conclusions drawn from the correlations already discussed.

Number wrong on quizzes.

Number wrong on papers.

	0	1	2	3	4	f
4	1	8	16	33	32	90
3	1	4	12	25	11	53
2	6	10	39	26	13	94
1	2	46	29	22	26	125
0	94	67	44	18	28	251
f	104	135	140	124	110	613

[2] The passing mark at Mount Hermon is 60.

The Mathematics Club of the Pontiac High School

By Margaret Steward
Pontiac, Michigan

The purpose of the Mathematics Club of the Pontiac High School is to develop an appreciation of mathematics, a greater interest in it, and a broader understanding of the subject than can be secured in the regular classwork.

The only unique feature of the organization is that members sign up for a semester at a time, a new roster being made out each semester. The only requirement for membership is a passing grade in mathematics for the preceeding semester, if the student was enrolled in a mathematics class. Fully half of our upperclassmen are no longer taking mathematics, but are sufficiently interested in the subject to continue in the club. We have found it necessary to limit our membership to 80, in order that all may be able to see any charts used, or demonstrations made at the board. This is the fourth year of the club, and at no time has our membership gone below 75.

The social activities of the club are limited by school ruling to one party and picnic per semester, thus eliminating those who would join the club for entertainment only.

The main feature of the club is the regular program, which is given every fourth Wednesday, during the activity period, 45 minutes in length. The transaction of necessary business occupies from 10 to 15 minutes, leaving about a half hour for the program itself, which is given by the club members, with the exception that on one program each year, we plan to have an outside speaker. These programs have covered quite a wide range of topics, most of which fall into one of these three groups:

(1) The history of mathematics.
(2) Mathematics in industry and art.
(3) Occasional plays or mathematical contests.

Doubtless I can illustrate this best by outlining briefly a few of our programs. From the first group, the history of mathematics—

always endeavoring to disprove the prevalent attitude that mathematics is static—we have used the following topics for programs.

1. Notation systems, showing the following:

   Cuneiform numbers, Babylonian.
   Heiroglyphic numbers, Egyptian and Chinese.
   Alphabetic numbers, Greek and Hebrew.
   Symbols, using both additional and subtraction, Roman.
   Symbols, using units of different orders, Mayan.
   Ten digits, using "O" and local value, Hindu-Arabic.
   Great stress is laid on the reason for the superiority of the Hindu, resulting in its almost universal use.

2. Methods of computation:

   Multiplication and division in Egypt, using unit fractions.
   In the Middle Ages, the "jalousie" method of multiplication and the scratch method of division.
   The abacus, as used in antiquity by the Greeks and Romans, and at present by the Chinese. Problems may be worked for the group on a simple model of the abacus.
   The Burroughs, comptometer, etc., as modern mechanical means of computation.
   The slide rule.
   This program correlates well with the preceeding one, showing the dependence of computation methods on the notation system used.

3. Development of standard units of measure:

   Origin of the prevalent units of length, weight and volume: their standardization.
   History and advantages of the metric system.
   Preservation of international standards; the function of the Bureau of Standards at Washington, D.C.

4. Time: its computation and measurment.

   Meaning of sidereal, true solar, mean solar, and standard time, how they are obtained and checked.
   Development of time-recording, from the simplest sundials of primitive peoples, sun-rings, waterclocks, sand-clocks (hour glass), through the clumsy but ornate clocks of the middle ages, to the modern, accurate clocks and watches in every-

day use. Models of many of these timepieces can be constructed simply, to make a little known story very fascinating.

5. The history of the calendar:

   A study of primitive calendars, showing how the irregularities of our present calendar originated.

   The proposed change to the 13-month calendar, showing both its advantages and disadvantages.

   At the time of the adoption of the Gregorian calendar by the Chinese, last January, a study of their former (lunar) calendar was made. This was very different from any other ancient calendar which has persisted over a long period.

6. Mathematics in discovery:

   In astronomy, the forecasting of the reappearance of comets, and the discovery of the planet Neptune by Adams and Leverrier bring out the use of higher equations and their graphs.

   In chemistry, the discovery of new elements by the mathematical periodicity of the Mendeleef table brings out a wholly different field.

   In engineering, such achievements of engineering accuracy as the Moffat Tunnel, the new Cascade Tunnel, and the Hudson Tube emphasize the marvelous precision possible in surveying.

7. Treatment of equations:

   A study of the Egyptian method of solving fractional equations using unit fractions, of the Greek solution of quadratics by a geometric construction would make any freshmen rejoice that he studies his mathematics in the 20th century.

8. Occasional biographies of great mathematitians emphasize the personal side, and even an underclassman can get some ideas of the questions raised by Einstein.

From the second group, that of the relation of mathematics to industry and art, one of the most easily developed is the subject of graphs.

1. We have illustrated the use of graphs by:
   a. A speaker from the Wilson Foundry, who brought a set of blue-printed graphs, such as is given to each foreman in the plant every two weeks.
   b. A set of banking graphs.
   c. Graphs made by an automatic recorder in a dairy.
   d. Weather bureau records.
   e. Road construction graphs.
   f. Forecasting of future needs in the auto industry by means of a periodic graph.
2. In the field of transportation there are several topics:
   a. Relocation of aërial trade routes along great circles.
   b. The construction of an airplane, its instruments, and the use of "dead reckoning."
   c. In ocean navigation the taking of latitude, longitude, and time, steering a course, present distinct problems of mathematics. Since the average high school pupil knows practically nothing of an ocean going ship, the housekeeping aboard ship and the allocation of space on a ship makes a very interesting problem.
3. The relation of mathematics to music is brought out in time, relative frequencies of the notes of a scale, length of strings and pipes in the instruments.
4. In art, geometric designs are found in domestic decoration, natural crystals and leaf patterns, windows and other conventionalized designs.
5. In architecture the ideas of proportion and balance, the psychoology of style and the like may be very well brought out if pictures can be thrown on a screen. A series of pictures of such buildings as the city library, Masonic Temple, Maccabbee building, and Union Trust building in Detroit, or the modern cathedrals of America now under construction, illustrate these very well.

In the recreational type of program might be noted:

1. An original playlet showing the catastrophic results which would follow the abolition of mathematics.
2. A farce representing a geometry class in which the memory is trained at the expense of the reasoning powers.

3. Spelling contests, construction contests, the unscrambling of mathematical words, solution of puzzles such as letter division and multiplication, catch problems in arithmetic, and so on.
4. Finding partners at a party by giving one group equations and the other group the solutions, each equation to find its solution.
5. At our banquet last year the perfection salads were cut into small triangles, circles, trapezoids, and the like, each plate receiving at least 3 designs.
6. The club sponsored the movie, "Animated Geometry," for the benefit of the geometry classes.

We try very hard to follow up any newspaper or magazine clippings pertaining to mathematics, occasionally running a current events feature. For instance, this fall, while Lindbergh's aërial exploration of the ruins of the ancient Mayan civilization in Central America made this a front page feature, we put in a program on this civilization. A general history of the race, with a study of their numeral system, calendar and style of architecture, illustrated by pictures thrown on the screen, met with a hearty response.

The club has been very active in stimulating interest in the state Academic contests at Mt. Pleasant, 11 of the 12 points won in mathematics, as well as some other points, being won by club members.

One of the great difficulties in planning programs is in securing variety; also in keeping the topics within the comprehension of the underclassmen, this latter being a serious handicap in attempting to discuss the modern research work in mathematics. It certainly keeps the teachers sponsoring the club from having too many idle moments, but it has seemed to us decidedly worth while. At least, the fact that the Mathematics Club is regularly the second largest club in school shows that mathematics is not so dead as many teachers of other fields—and unfortunately some in our own field—seem to believe.

Prize Problems for Prize Pupils*

By Elbert H. Clarke
Hiram College
Hiram, Ohio

The public schools of our country have been undertaking a tremendous task. With limited means, and with more pupils per teacher in many instances than is really wise, they have boldly set about educating all of the citizens of our democracy. And, considering all handicaps, they are doing remarkably well. For the boys and girls and the young men and women of the masses there has never been such a time of opportunity in all history for intellectual development to the very limits of their abilities.

But have we teachers been doing all that we could for the youth whose ability ranks him in the upper fifth or in the upper one per cent? Have we recognized the fact that his mind needs stimulation too? Have we not comforted ourselves with the assumption that somehow, sometime, he will find himself and naturally slip to the front? But is this not the denial of values in our teaching? If the slow and average mind is stimulated to do more than it would otherwise do are we to be content? It would be as if a farmer spent all of his time and money in bringing barren fields into production while the richest spots on his farm were left to weeds and brambles. America's greatest undeveloped resource is the brain power of her citizens. The intellectual keenness of the top twenty per cent means more for the wealth, the power, the greatness, and the goodness of our country than all of the bustling activity of the remaining eighty, and it is a fallacy to assume that the latent energies of this group need no stimulation. The trouble is partly with us teachers. We are often in the presence of ability superior to our own and do not know it. Our problem should be essentially this—without neglecting the lower four-fifths, how can we keep the upper one-fifth usefully busy in getting not only the education of the average man, but also the training of the superior man?

*** Slightly condensed form of a paper read before the Mathematics Department Meeting of the Northeastern Ohio Teachers' Association, Cleveland, October 25, 1929.**

It would seem that we who teach mathematics are in a position to answer this question most easily. For mathematics is more than most of us have usually taught our classes to believe it is. It becomes clearer every year that a mathematical form is the goal of every science that makes any claims of exactness. Long ago it was conceded that astronomy and mechanics were methematical in their nature. But I suppose that Benjamin Franklin lived, experimented, and died without once guessing that the future would put the whole theory of electricity on a profoundly mathematical foundation. Chemistry is moving more and more in the direction of exact mathematical methods in its theories. Some very interesting work is being done today in mathematical statistics with very important applications to the biological and social sciences. If any of us are teaching students who expect later to have anything to do with work calling for precision of measurement or exactness in grouping or ordering of data we can insist with far greater backing of authority than was possible even ten years ago that a thorough mathematical preparation is necessary.

The superior mind is active in terms of problems. Just as the hero of a detective story is collecting, weighing, testing, sorting, ordering every scrap of evidence and every thinkable hypothesis in order to complete a clear and convincing picture of the crime so the scientific investigator works on his problem, carrying it around with him wherever he goes, even taking it to bed with him at night. You will recall the story of how Kekule suddenly *saw* the atoms of the benzene molecule join hands and form a ring as he lay half asleep after a long period of intense study and effort to solve this great and fundamental problem of organic chemistry. This brooding over ideas until the mind is saturated with the details of the problem it wrestles with is not peculiar to the exact sciences but is the mental habit of every great thinker. The mental habits of a Pasteur or a Darwin are not essentially different from those of a Galileo or a Newton; they are the mental habits of a solver of difficult problems.

Mathematics, in one aspect, is problem solving, and its whole history might be written as the story of a few great problems and their posterity; and I lay it down as a fundamental article of belief and a main reason for the presentation of this paper that the superior type of mind can build the problem solving technique and habit most easily and naturally by the use of mathematical problems.

Our modern textbooks contain excellent problem material—for the

lower 80 per cent. The problems are likely to be well graded, they have been tested for special values and difficulties; as drill work they will not need much further improvement. But of problems that demand high concentration and that brooding over ways and means of attack which marks the able thinker we have almost none. Perhaps it is just and proper that it should be so. Some of us might have no better sense than to condemn a pupil for not being able to do thinking that was beyond his means. But can we not give to Tom the stimulus that he needs without forming at the same time an inferiority complex for Dick and Harry? If our present methods of teaching cannot provide for this, so much the worse for our methods. Perfect teaching will feed the mind and fire the ambition of every pupil.

For several years I have been trying out these ideas with my college freshmen. (It is my belief that a college freshman—at least for the first semester—is simply a high school student who has had a summer vacation.) For instance, very early in the college algebra, while in the process of reviewing elementary principles, we do a good many mental problems in squaring numbers, and using the difference of two squares for finding numerical products, and so on. Then I like to give them the well known rule for squaring numbers ending in 5 and ask them to try to give a written and reasoned account of why it works. I find that almost nobody has ever observed that the second differences of consecutive squares are constant. I suggest that there must be some simple explanation of that fact too. Somebody is almost sure to ask if the same thing is true of consecutive cubes. "All right," I say, "go to it and find out!" Or, just a little later we have mentioned the rule for divisibility by 9. A problem or two on the changing of the digits of a number generally gives a clue for the proof, for small numbers, of this rule. I have found that my freshmen can become almost excited over the fact that our scale of ten is not the only way in which numbers could be written. To see the operations of arithmetic carried out correctly for any numbers with a scale of two and no digits other than 0 and 1 is quite a new thing. A number of people who are of average ability will have an interesting time writing numbers on various scales and incidentally learning a great deal about our system of integers. These are not the only problems that are given. There are many problems too difficult for all the class but which all can understand when demonstrated. And then there are problems which do not demand brilliance of mind

so much as clearness of mind and steady hard work. These two qualities are of more importance than brilliance in the making of keen business men, professional men and teachers. I think that problems involving successive approximations to a numerical result, supplemented by graphical methods, are excellent material of this type. The problem of finding the angle of a segment equal to one-third of the area of a given circle is such a problem. Once in a while a problem involving painstaking and careful computation is valuable. I remember being called away from the breakfast table by an enthusiastic student who had been working all night computing pi to twenty places. He had the twenty places right too, but that aspect of the problem added nothing to the stock of human knowledge. However, I have not the slightest doubt that the knowledge that he could and had performed a task involving the elements of hard and accurate intellectual work has been of immense benefit to him in facing hard thinking in the years since. There are problems of this type in Geometry, where very careful and accurate constructions are required.

I do not believe that we should encourage the brilliant pupil to run ahead of the class. Possibly it is inevitable that he will do it. I know that one of the virtues claimed for laboratory geometry is that it enables the rapid worker to finish plane and solid geometry while the bulk of the class is doing the required plane geometry. I have my doubts about the wisdom of this procedure. Every branch of mathematics is so broad that a vast amount of material remains untouched in any course whatsoever. It is an absurd and vicious notion in any course from the elements of arithmetic to the highest unearthly abstractions of a graduate seminar that the whole possible subject has been covered and completed. It is equivalent to the assumption that the text and the teacher, between them, know it all. Sometimes this impression is conveyed innocently enough, but the real teacher will always leave his pupil, not with the feeling of being "fed up" with that particular kind of information, but with a keen desire to go voyaging upon the strange seas of thought now glimpsed for the first time.

In what I have been saying I have assumed that there is a certain amount of material in any course which must be covered and that there are students in most classes who find the routine work uninteresting, or at least unexciting, and who are left with nothing to

do after the required lessons are finished. It is they who should be given the extra stimulus. It is they who are to become, I believe, the great educational problem of the near future. We teachers of mathematics have the privilege of furnishing the intellectual stimulus that shall produce, not only the comparatively few great mathematicians the next generation shall need, but the physicists, the chemists, the biologists, the statisticians and economists; and yes, the political, social, and religious leaders of clarity of thought and vision that we shall need so desperately. For where is there any task in all these great fields that does not call for exactly those qualities and habits of keen thinking, of utter impartiality as to hypotheses, of rigorous weighing and testing, and of profound imagination which must, in some measure, be part of the mental life of any successful solver of mathematical problems?

Notice to Subscribers

BEGINNING WITH THIS ISSUE THE MATHEMATICS TEACHER will be printed by the George Banta Publishing Company, of Menasha, Wisconsin.

We have changed the magazine in many respects, introducing new ideas which will give added life and interest to it without detracting from its dignity or readability.

To begin at the very front we want to call your attention to our new cover in which we feel a justifiable pride. It is done in a manner forceful and firm, befitting the dignity of the world's most exact science. The type used is Kabel Bold, a new face just imported into America from Europe, and we have used a sketching of Archimedes, one of the earliest and most famous mathematicians. We have added a contents and title page and a frontispiece, thus adding to the beauty of the publication. A larger headline type has been introduced, which adds to the readability. Kabel Bold type again has been used for the headlines, while at the end of each article we have introduced a small decorative cut which relieves the monotony of solid pages of type. The new dress was designed by Leland F. Leland of the George Banta Publishing Company.

We hope you will like the new MATHEMATICS TEACHER and we shall be especially pleased to have your comments.—THE EDITOR.

Will you come to see our mathematics when you come to the exhibit on March 9^{th} to get a ______ on geometry and see our work from a different $\angle$?

We will take you into a new world - Flatland - and show you its inhabitants. There you will see Roman floors and Gothic windows. We have even tried to capture all the snowflakes so that March 9^{th} will not be stormy.

Here are formulas to show you how to get around this city $(c = \frac{h}{k} + s)$ and to tell you what will get you about at the smallest cost $[c = 15 + 20(m-1)]$

Pleasures are provided by hurdle races in which the unwary gets behind because she does not regard the signs

Even your vacation trip is planned since we can start you on your way to any star you wish, though by the maximum rate of calculus you will not get there very soon.

These and other things are in our land.

Won't you come?

Mathematics Classes
First Year Committee

The preceding poster was prepared by a group of seventh grade children in the Horace Mann School for Girls in New York City to explain the nature of a mathematical exhibit to be held in the school and to induce parents and pupils in the school to see the exhibit. We think it will be of interest to our readers.

Remedial Work in High School Mathematics

By L. H. Whitcraft
Ball State Teachers College, Muncie, Ind.

Introduction

Teachers of high school mathematics are confronted with the fact that there are more failures in the mathematics of the secondary school than in any other subject in the secondary school curriculum. These failures may be traced to some one of the following factors; (1) the materials of mathematics, consisting of the textbook, practice exercises, and special devices; (2) the teacher's method of instruction and manner of presenting the subject matter to the pupils; or (3) the methods and processes of the pupils themselves. Now that the teachers of mathematics realize that there is a great amount of criticism due the department of mathematics what are they going to do about it? The answer should be the same as the elementary teachers have given to the criticisms which have come to them—give remedial work.

What Remedial Work Is

Remedial work in high school mathematics may be compared to remedial work in the field of medicine. When one becomes sick the family physician is called. He diagnoses the case and then begins to treat the patient for that particular ailment. If one has a bone out of place the physician will put it back in its proper position and the defect is remedied. On the other hand if the bone be broken it is set in its proper position and firmly maintained in that position until the broken ends have grown together or knitted as is commonly said. Sometimes neither of the above things is the trouble. It may be the diagnosis will show a lack of nourishment, a lack of exercise for certain muscles, or an over amount of exercise. In any case a thorough examination points out the trouble and some type of treatment will be suggested which will remedy the trouble.

In high school mathematics something similar to the above may be causing the pupils a great deal of concern. They see that they are not doing as well as the other pupils are doing and yet they do not know what the cause is or how to overcome it. It, therefore,

becomes the duty of the teacher to find out just what these things are which keep the pupils from progressing as rapidly as they should and after this has been done to suggest remedies and also administer them. The particular difficulties of the pupils may be found by giving diagnostic tests on those processes or operations in which the pupils are having trouble; by observing the pupils at work; or through conferences with the pupils. That which is done to overcome, to correct, or to eliminate any of the various difficulties which the pupils are having is the remedy and the material used is known as REMEDIAL MATERIAL.

Reasons for Remedial Work

There are a good many reasons why remedial work should be given. Among these reasons are the following:

(1) *Low grade learning ability of the pupil.* In almost every high school we find one or more pupils whose I.Q's are low and some of these pupils get into mathematics classes. When new work is taken up and developed these pupils do not get it mastered well enough, until it has been retaught or redeveloped two or three times, to proceed with the assignment of the lesson. This reteaching is the remedial work.

(2) *Incompetent teaching.* There are a great many poor teachers in the secondary schools, teachers who do not know how to develop a topic or a lesson as it should be developed or who are too lazy to exert themselves to do a fair job of it. High school teachers complain about the products of our elementary school and the universities and colleges in turn complain about the products of the high school that come to them. Many times pupils are found in the advanced courses of high school mathematics who have been poorly taught in the prerequisite courses. Before these pupils can do anything worth while in the advanced courses they must have been re-exposed and retaught all those facts, principles, and processes which were poorly taught them.

(3) *Textbooks are poor self-teaching texts.* This is especially true of the textbooks in secondary school mathematics which have been on the market for several years and is generally true of the new texts. The texts are especially weak in the following points; (a) the language is not that of the child but of the adult and the pupil does not understand what is meant; (b) the material is not well

developed or rationalized; (c) the model examples do not always illustrate the various difficulties of the exercises which follow; and (d) there is no statement given or means suggested in the texts by which the pupil can know just how well he is doing or what he should do to bring about the needed improvement which he feels he should have. Teachers, therefore, must give a lot of work to make up for that which the textbook teaches poorly or not at all, and, in addition, make it possible for the pupils to know just how well they are doing at any time in the course. Teachers will look forward to the time when high school texts in the field of mathematics have remedied the defects which have been shown to exist in their makeup. This time will probably not be far off, for the texts in the process of development today have taken the aforesaid defects into consideration.

(4) *Hard-spots which need an extra "going-over."* Many of the most difficult parts of algebra and geometry need a second or maybe a third going-over before they are completely mastered. Some examples of these hard spots in high school mathematics are subtraction of signed numbers, exponents, factoring, ratio and proportion as presented in most geometries. The recalling of these difficult ideas systemmatically offers to the pupil increased chances for genuine understanding and helpful guidance.

(5) *To place special emphases upon the tools of high school mathematics.* The equation and the formula which are the main tools of high school mathematics need to be given special attention in order that their importance may be impressed upon the minds of the pupils. Frequent drills upon these two phases of mathematics are essential.

(6) *To deepen and strengthen a first partial mastery.* Many processes in high school mathematics are long and complicated and it is almost impossible for the student to gain complete mastery the first time over the work. It, therefore, becomes necessary to give remedial material to strengthen that which the student has only partially mastered.

(7) *To repair poorly learned things in mathematics.* Pupils, frequently, find when they attempt to apply the principles which they have learned that they have only partly learned them and that before they can do their assignments well they must study further the principles involved.

(8) *To counteract superficial self-complacency.* Bright pupils

are usually superficial. Many times they believe that they have thoroughly mastered some new bit of knowledge for they can apply the new knowledge immediately after it has been learned. However, in a short time they find that they are unable to do that which they believed they could. Some remedial material is necessary to firmly fix this new knowledge in their minds.

(9) *To thoroughly master highly important items.* In every high school mathematics course there are some important items which must be gone over again and again in order that the pupil may always have them at his finger tips.

(10) *To make surveys of real value.* At the end of a semester's work in either algebra or geometry it is highly important that the teacher take an inventory or make a survey of the facts which the pupil has at his command to see if he has a sufficient amount of knowledge to take up the work of the next semester. Should he be deficient in any particular thing he should be given remedial material which will overcome this deficiency.

(11) *To counteract disturbing influences of subsequent and analogous topics.* "A process when mastered alone is not sufficiently understood until it has been mastered in different situations. It is evident that when a new process is taught the previous processes involved offer new difficulties and deserve considerable attention. Such a situation is met when arithmetical computations occur as in an algebraic problem. Errors such as $2x \times 3x = 5x^2$ are common. Certainly a pupil knows that 2 x 3 is 6, but he must now concentrate on two processes at the same time and he must learn to do both correctly."[1] A review on arithmetic alone will not help him avoid such mistakes.

(12) *Needed between a first and a second exposure to a final mastery.* After a topic has been developed there should be some material given which will fix the points to be remembered or understood, in the minds of the pupils. According to the laws of learning this drill should be given after the first exposure of the topic to be learned and the second exposure and is remedial in its nature.

(13) *To reteach a topic that has been poorly taught or not taught at all.* One of the very first things a teacher should do in taking up a new topic is to motivate it in some way. A failure to do this calls

[1] Breslich, E. R. Mathematics Teacher, 14, pp. 276-291.

for remedial material which will aid in selling the topic to the student.

(14) *To recognize individual differences in attitudes, aptitudes, and appreciations.*

(15) *To complete any diagnostic program.* Absolutely no good can come from learning the types of mistakes pupils make or the difficulties they have in mathematics unless there is an attempt to overcome them. Diagnostic tests find the difficulties and the remedial material overcomes them.

(16) *Every individual has one power greater than any other and it should be developed.* The teacher should try to find the one power which a pupil has greater than any other. In most cases this power is not difficult to find. After it has been found the pupil should be given every opportunity to develop it. This is done by giving him drills and problems in which he applies it and also by providing him with literature upon the topic.

(17) *To fight forgetting.* "Most students forget, not only the minor and unessential facts, but the most fundamental, the most necessary ones, so completely that the returns for time and labor spent seem to be wholly inadequate."[2] Concerning the necessity of fighting forgetting I quote the following,[3] "After a skill has been mastered, and the class has moved on to new topics, some means must be taken to prevent the pupil from forgetting, through disuse, skills which have been painstakingly built up. Nothing is more discouraging to the teacher than the discovery, after an interval of a few weeks or months that her pupils have forgotten most of the instruction she has previously given. No matter how efficient the original teaching, rapid forgetting is an inevitable law of psychology unless definite means of combatting it are systematically pursued."

The surest way to prevent forgetting is to fight it week by week by spending a little time on remedial material which contains examples and problems involving previously taught skills.

Character of Remedial Work

In selecting material of the remedial type teachers should keep in mind the following essential features:

(1) *The material should be selected which will bring about cer-*

[2] Schultze, Teaching of Mathematics in Secondary Schools, p. 4.

[3] Knight, Studebaker, Ruch—Teachers' Manual for Work Book, Grade 5, p. v.

tain definite ends. Thorndike has emphasized this particular point in his chapter on drill in arithmetic in "Newer Methods in Arithmetic." Whenever drill is to be given it should be given for a certain definite purpose and not the hap-hazard type which has been so common in the high schools.

(2) *It should be on component elementary skills.* "One of the most frequent failures in the teaching of algebra is due to the tendency to see an objective as a single entity, when as a matter of fact it may seem to the pupil very complex. The solution of a mathematical problem is rarely an exercise of a single skill; it almost invariably requires not merely several skills but the ability to coordinate them. To take an example from arithmetic, a child may have perfect mastery of the six skills involved in 4 x 5, 4 x 8, 4 x 2, 6 x 5, 6 x 8, 6 x 2, besides those involved in simple additions, and still be unable to multiply 582 by 46, an operation which requires precisely these elements. He fails in neglecting to drill upon it.

Similarly in algebra, we not only need to see clearly the great objectives but we need to analyze each into unit skills and then be able to coordinate these on a working basis. The teacher should see that the pupils be given a chance to make every possible error that can occur in attaining the objective under consideration. In this way his deficiencies can be discovered and the remedy applied. This remedy consists in sufficient practice, particularly in the region of the defects, to insure satisfatory mastery of the process."[4]

(3) *It should be largely self-administered.* It is a very good plan to provide material by which a pupil may take some responsibility for his own learning and at the same time relieve the teacher of one of the many duties which she has to perform. This material could very well be made for home study.

(4) *It should be standardized with some flexibility.* "The motivating effect of standardized drills depends upon one of the most powerful incentives to learn—the awareness of success at the moment of learning. Probably no other condition affects the rate of learning more favorably than practice undergone with full knowledge of the gains made by the learner."[5] At the present time we do not have standardized drill exercises for either algebra or geometry. Teach-

[4] Smith and Reeve—The Teaching of Junior High School Mathematics, pp. 197-198.

[5] Knight, Studebaker, Ruch—Teachers' Manual of Work Book, Grade 5, p. vi.

ers will, therefore, use what material they have at hand and in many cases arrange their own material for remedial work until drills are obtainable which have norms worked out.

(5) *They should be administerable to a class, to individuals, or to groups.* Remedial material should be so organized that it can be given to an entire class when the class as a whole is deficient upon the same thing. This usually does not happen for there are in most classes a few students who do not need the same type of work as the majority of the class. In this case these students should be given that which they need. Hence the need for the type of material which can be administered to individuals. When there are several in the class who need a particular remedy they should be given it as a group. It is, therefore, necessary that remedial material be so organized that it may be given to a class as a whole, to individuals of the class, or to small groups in the class.

(6) *They should be correlated with the most widely used tests.* At present we can scarcely say that there are any very widely used tests in high school mathematics. I have inquired of a good many high school teachers from various high schools concerning this very fact and not one of them ever used any tests other than those of their own make. Until we do have widely used tests we had best arrange the remedial material so it will correlate as much as possible with those tests which appear in mathematical journals.

(7) *They should be correlated with the textbook.* In so far as is possible all remedial material should be so organized that it will correlate with the text used. This will eliminate all confusion concerning the material covered in the remedial work and not covered in the textbook.

(8) *Remedial material should be provided with answers.* As was stated in an earlier paragraph one of the greatest incentives for learning is the awareness of how well one is doing. In our remedial work, then, we should include all those things which make for a greater degree of learning. In other words there should be answers to remedial material whenever possible.

Determining Where Remedial Work is Needed

Before remedial material can be given we must first locate the particular difficulties which need it. There are various methods by which we arrive at the conclusion that some reteaching is necessary

or that additional drill should be given. Among these are the following:

(1) *The inventory test.* This particular type of test gives a general survey of the knowledge which a pupil has and it will point out the type of problem in which lies the difficulty but it will not point out the specific element in that problem which is causing the trouble. This is an excellent method of locating the difficulty in a general way and it will show that some sort of remedial material needs to be given. In order to find what this remedial work should be it becomes necessary to give another type of test, the diagnostic test.

(2) *The diagnostic test.* The primary purpose of the diagnostic test is to find out why any particular pupil is unable to succeed. Without some plan of discovering the particular defects of a pupil or a class the work of the teacher is likely to become more or less futile. The diagnostic test tests the pupil on each component part of a unit and in this manner, those difficulties which the pupil has will be definitely located. When this is done the special type of remedial work needed can be administered.

(3) *By observing pupils at work.* The supervised study period is the place where the teacher locates the majority of the difficulties which the pupils have. By walking about the room and observing what the pupils are doing the teacher readily sees, many times, what is bothering the pupil and at the same time gives the student helpful advice or suggestions.

(4) *Through conferences with the pupil.* Another excellent way for the teacher to determine upon what the pupil needs remedial material is through the conference with the pupil. All teachers, especially those of high school mathematics, should set aside some definite time during the day when pupils can come with their problems and discuss them freely. An hour a day is not too much to expect of the teacher for this type of work. Many pupils are too backward to ask questions before the entire class but they will come to the teacher at another time and talk freely concerning their troubles. This conference should be given over wholly to remedial work.

(5) *Self diagnosis of difficulties.* Many times without any suggestion from the teacher the pupil will discover that he needs to restudy some particular topic before he can go far toward mastering a new topic and he will take it upon himself to review or restudy the necessary facts.

(6) *Through contests.* In contests where both speed and accuracy count pupils many times find that they need much drill on certain processes and facts in order to make a showing as good or better than the other contestants.

(7) *Reading the lesson orally.* There are times when the teacher may locate the difficulty by having the pupil read the lesson orally. These difficulties are usually in difficult or unfamiliar words and poor sentence structure.

These seven methods given will locate the difficulties which the students encounter in their work. The particular type of remedial work given will depend upon the difficulty found.

Theoretically every error of every pupil should be a matter of study on the part of the teacher. Practically, however, purely individual teaching is seldom feasible under actual school conditions. The compromise position suggests that the teacher direct her remedial work to those examples and problems most often missed, extending the discussion to the less troublesome points as far as time permits.

Common Errors Which Call for Remedial Work

A study of the common errors in ninth grade algebra was made by Miss Virginia Wattawa in the East Side High School of Madison, Wisconsin. This list of common errors is similar to other lists made of the same subject. Included in it are the following:

a. Errors in fundamentals of arithmetic.
b. Errors in signs in removing parantheses.
c. Errors in signs in addition and subtraction.
d. Errors in signs in multiplication and division.
e. Errors in clearing of fractions.
f. Errors in extracting square root.
g. Incorrect operation.
h. Errors in copying work.
i. Errors in factoring as $(a - 2)^2 = (a - 2)(a - 2)$.
j. Errors in forming the equation.
k. Errors in completing the square in quadratics.
l. Errors in use of exponents.
m. Errors in solution of equation. (Unknown term given in answer).
n. Failure to perform the same operation upon both members of the equation.

o. Lack of knowledge of words.

p. Incomplete solution.

q. Failure to extract root when removing the radical sign.

This list would have been longer had the study been carried over the entire year instead of just a period of three months. However, this list does suggest the type of errors which pupils made and upon which we as teachers must focus our attention in order to correct them. The noticeable thing about this particular study is the fact that out of 407 definite errors recorded 144 or 35.4% were errors in simple arithmetic; the next largest number of mistakes, 76, or 18.7% were errors in signs; there were 44 errors or 10.8% in copying and reading; 10.3% in the incorrect operation; and 9.3% of the mistakes were in the lack of understanding of terms. These five types of errors constituted 344 of the errors out of the 407 mistakes listed or 85.4% of the total number of mistakes. This simply means that most of our remedial teaching for the beginning class in algebra should be directed toward correcting these few types of mistakes.

Remedial Material

Material which has been assembled for the purpose of overcoming the common errors made in high school mathematics is of the following types:

(1) *Drill exercise.* One of the chief types of remedial material is the drill exercise which is given to students to habituate certain facts; to increase the rate of speed of performing particular operations; to overcome careless habits; to increase accuracy in operations; and to quickly review type forms. These drill exercises are found in some of the recent texts in high school mathematics either distributed through the text where they are first likely to be needed or placed in the supplement where they may be easily found if needed. The authors of present day texts in arithmetic have gone far to remedy the common errors which children make in arithmetic by placing a great many timed drills through the books in which norms have been worked out. The children can, therefore, by doing these drill exercises or tests know how well they are doing. In addition to these drill exercises these same authors have placed in the back of the arithmetics much material which will help the pupil over the hard spots.

Nothing like the above program, found in the recent texts in

arithmetic, is found in our texts in algebra. It may be that we do not need such an extensive program for remedial work but we do, surely, need some material similar to that for both the algebra and geometry work. Teachers of high school mathematics should have at hand drill material for each type of mistake the pupil is likely to make and just as soon as this mistake is located the material should be given the pupil and he set to work to correct it.

(2) *The drill card.* Many teachers prefer the drill material arranged upon cards as that is an easy way to keep it and give it to pupils needing it. A complete list of 130 sets of 10 cards each on algebra may be obtained from the Scott Foresman & Company. These were made by John DeQ Briggs. The examples of each set are fairly even as to difficulty. When a topic is important enough to require two or more sets of cards, the successive sets are progressive in difficulty. For example there are seven sets on affected quadratic equations ranging from easy examples in type form with rational roots to hard literal quadratics. There are occasional review sets and some very hard sets which are meant for the better pupils only. Such drills as presented on these cards offer an excellent review at the end of the term or semester. A weak pupil will profit greatly if he will spend some time on these cards and he will usually like such work. If the pupil shows weakness on a particular type he may work out the entire set on that topic with the extra incentive of knowing that he may meet one of the same type in the class room.

(3) *The flash card.* The flash card type of drill is especially desirable for hasty reviews on fundamental facts and also for fighting "forgetting" of these facts. A few minutes given now and then in drill of this type will do much to habituate those knowledges in high school mathematics which are desirable.

The flash card may contain drill upon almost any topic in either algebra or geometry. An example of this type in algebra is to arrange each of the following statements on a card which is to be exposed to the pupil or class a few seconds and then have the pupils write or give orally the answer.

If $4x=20$, $x=?$	If $7x-3=11$, $x=?$
If $15=3x$, $x=?$	If $9-x=2x$, $x=?$
If $2x+x=12$, $x=?$	If $10+x=3x$, $x=?$
If $3x=8-x$, $x=?$	If $5x-3x=16$, $x=?$
If $5x+2=17$, $x=?$	If $4x=25-13$, $x=?$

The cards upon which these statements are written should be large enough so that the statement would not be crowded and yet at the same time could be seen from any place in the room. Many sets of cards similar to this on various topics of high school mathematics could be used to good advantage. It would, therefore, be extremely worth while for the teacher to spend some time in working up material of this nature to be used throughout the course.

(4) *The work-book.* The Standard Service workbook in arithmetic has for one of its purposes, the providing of drill material to prevent forgetting and for another to provide silent reading lessons on important ideas of arithmetic that every child, for that particular grade, should keep in mind along with actual skill in computation. It is as useful to review systematically the thinking involved in high school mathematics as it is to review computation. The recalling of difficult ideas systematically offers to the pupil increased chances for genuine understanding. A workbook for algebra or geometry similar to the one mentioned above in arithmetic would be a great asset in the teaching of high school mathematics. In so far as I know there is no such workbook on the market.

(5) *The text itself.* A great many teachers use the same material in the text for remedial work which they used for first teaching. A regoing over of the work once covered will do much, many times, to set the student right on some particular thing which he did not fully understand the first time over the topic. This is especially true in the subject of geometry.

(6) *Extra problems.* It is a common practice of some writers of textbooks to distribute through the text "extra problems" which are to be used by those pupils needing additional work. This is a type of remedial material and should be used as such.

(7) *List of principles or rules to follow.* Sometimes when pupils get into trouble the best method of aiding them is to give them a set of principles or rules and have them follow them. This is especially true in case of inability to solve verbal problems. The remedy for this inability to solve problems of this type is a general method. The pupil needs a method to show him how to begin in every case, and some additional suggestions which he may apply to particular types of verbal problems. These directions should be so definite that the teacher should get immediate action from every pupil the moment a problem is assigned. Suggestions such as (a) In many verbal prob-

lems the equation is obtained by translating the statement word for word into algebraic language; (b) The facts involved in verbal problems are usually related to each other.

In addition to these two suggestions there should be given specific directions for the student to follow in problem solving. These directions are:

(a) Read the problem carefully.

(b) By further reading find out what the problem asks for.

(c) Let some letter, as x, denote this unknown number.

(d) Read the problem again, one sentence at a time and express the various facts involved in terms of the unknown literal number.

If the teacher will insist that the pupils who are having difficulty in solving verbal problems take these four steps every time, he will train the pupils continuously in expressing in terms of algebraic symbols, facts given in the form of verbal problems. The pupils will also get a start which usually results in a successful solution.

(8) *Referring to a model.* This is not a good method in every case but it is one which teachers use many times. The teacher will tell the pupil to turn to the model and study it when he is having difficulty. Often this does not help. Take, for instance, the following problem in the solution of fractional equations as a model for equations of this type.

$$\frac{x}{3} + \frac{x}{5} = 8$$

$$\frac{15x}{3} + \frac{15x}{5} = 15 \cdot 8$$

$$5x + 3x = 120$$

$$8x = 120$$

$$x = 15$$

Check

$$\frac{15}{3} + \frac{15}{5} = 8$$

$$5 + 3 = 8$$

$$8 = 8$$

A child follows this explanation through and then when he comes to the next problem which we say is $x/4 - x/7 = 6$, the first thing he does is to multiply by 15, for in the model problem 15 was used as a multiplier, or he may get the correct multiplier 28 and then have for

his first step $28x/4—28x/7=28.3$. The trouble was simply this, in the model he saw 15.8 and he considered that this 8 had been obtained by adding the 3 and 5. Therefore, in his problem he would obtain the number to be multiplied by 28 by finding the difference between the 4 and 7 since there was a minus sign between the two numbers. To overcome such difficulties, there should be more model problems in which there is a very careful selection of numbers and operations. A good model problem for the above type of equation would be $x/3+x/4=14$ then let the second model be $x/3—x/4=14$, and the third model be $2x/3+4x/5=44/15$. From a study of these model solutions the student would encounter the facts which he needs to know. Such models as these would make excellent remedial material.

(9) *Verbal aids.* Much remedial material is given verbally by the teacher in the supervised study as he passes from one student to another. This may be given in the form of a suggestion or a question but it is directed at the particular thing which is causing the trouble.

(10) *Contrasts.* In case there has been some disturbing element causing the trouble as when a pupil has just finished the topic of fractional equations and then comes in contact with a fractional expression to simplify, he invariably attempts to clear fractions. To remedy such errors from occurring again the two should be taken up together and the attention of the student called to the contrast between the two types of problems. A few examples contrasted will usually overcome the difficulty. Many similar cases from algebra can be cited.

Chart for Remedial Work

It is a very good plan for the teacher to make a chart or keep some sort of a record of the type of error the pupil is making and along with this what he should do to overcome it. The following are examples of the type of record which should be kept.

(1) J. S. was unable to translate any of the verbal problems into the proper symbols. He, therefore, must have drill in translation and need not concern himself very much with the solution of mechanical problems.

(2) C.B. was absent a great deal. He has not mastered the principles of removing parentheses especially when preceded by a minus

sign. Needs to be retaught the fundamentals involved and given drill in removing parentheses.

(3) R.F. is not clear on the sign of the product of two signed numbers. Has the tendency to put the sign of the larger number for the sign of the product. Needs to have contrasted the addition and multiplication operations and given drill. The flash card will suffice.

(4) O.M. understands the work covered by test and is accurate in fundamentals. Need not take any remedial work. May work on the advance assignment.

(5) A.T. understands the work covered and is accurate in fundamentals but is slow. Should be given timed drill and then asked to improve her record.

(6) J.H. adds denominators as well as numerators in addition of fractions. The terms of the fraction should be restudied. A few examples from the arithmetic dealing with simple fractions will help him get started.

Summary

To sum up briefly what has been said in the preceding paragraphs, the following statements are given:

(1) Students in high school mathematics make a great many common mistakes.

(2) These common mistakes can be definitely located.

(3) After the particular difficulty is located something definite should be done to overcome it.

(4) That which is given the pupil to do to overcome his difficulty is REMEDIAL WORK.

(5) The essential characteristics of remedial material are:

- (a) It should be selected so as to bring about certain definite ends.
- (b) It should be on component elementary skills.
- (c) It should be largely self-administered.
- (d) It should be standardized with some flexibility.
- (e) It should be of such a nature that it can be administered to a class, to individuals, or to a group.
- (f) It should be correlated with the most widely used tests.
- (g) It should be correlated with the text.
- (h) It should be provided with answers.

(6) Charts should be kept by the teacher showing the pupil's difficulties and suggesting the remedial measures to be taken.

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The Slide Rule as a Check in Trigonometry

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For some years the use of the slide rule as a check in trigonometry has been allowed in the Regents' Examinations of New York State provided that all the computation work appears on the answer paper. The State Education Department insists that the students of trigonometry be trained in the use of tables of natural and logarithmic functions. This training is tested by the requirement that all computation work appear on the answer paper. Having tested this part of the work, the Department encourages the student to check his work as any engineer would do—by the slide rule. The procedure is:

(1) *Solve the triangle by logarithms.*

(2) *Check to 3 significant figures by the slide rule.*

(3) *If time allows, check to 5 significant figures by the usual logarithmic check or the check of natural functions.*

There has been considerable experimenting as to the best method of using the slide rule in trigonometry. The following method has been adopted at Stuyvesant High School, New York City, as best suited to classes in trigonometry.

As early in the term as possible, preferably as soon as the class begins to solve triangles by natural functions, the work in the slide rule is begun. It is desirable for the student to have at his command the check by the slide rule as soon as he can use it.

Students are urged to buy their own slide rules. Since they will require the rules in college, they may as well have them in high school and save considerable time in the calculations of mathematics and science that occur in their home work, class work, and examinations.

If a student cannot afford to buy his own rule, the school has a certain number of slide rules which are loaned to the student for a week or two while the instruction is being given.

Teachers find that about five recitations devoted to intensive work are sufficient for teaching how to check triangles. The usual plan is:

Lesson 1. *Multiplication and Division.*
2. *Proportion and Simple Square Root.*
3. *Checking Right Triangles, using the Law of Sines.*
4. *Checking Oblique Triangles, using the Law of Sines (Cases A.S.A., S.S.A.).*
5. *Checking Oblique Triangles using the Law of Sines (Cases S.A.S. and S.S.S.).*

Regular home work in practice problems is required. After the presentation, speed and accuracy are secured by use of the slide rule throughout the term. On the mid-term examination, one question is on the slide rule. An adequate test on the rule should have a time limit.

Following is a detailed description of how the slide rule is used in the various cases of Right and Oblique Triangles.

Right Triangle

Given an Acute Angle and the Hypotenuse

Solve the right triangle given: (Wentworth and Smith Trigonometry, page 62)

$$A = 34° 28', \quad B = 55° 32', \quad c = 18.75$$

The logarithmic solution gives:

$$a = 10.61, \quad b = 15.46$$

Check:

$$\frac{c}{\sin C} = \frac{a}{\sin A} = \frac{b}{\sin B}$$

Substituting:

$$\frac{18.75}{\sin 90°} = \frac{10.61}{\sin 34° 28'} = \frac{15.46}{\sin 55° 32'}$$

Since the first ratio, $c/\sin C$ is known, we use this for the setting of the rule. The diagram is as follows:

A	*Opposite* 18.75	*Read* 106	*Read* 155
S	*Set* 1 (*sin* 90°)	*Opposite* 34° 28′	*Opposite* 55° 32′

Opposite 18.75 on the *right* half of scale A set sin 90° on scale S.
Opposite 34° 28′ on S read 106 on A.
Opposite 55° 32′ on S read 155 on A.
This checks to 3 significant figures.

To check the decimal point, note that the sides are in the same order of magnitude as their opposite angles and roughly proportional to these angles, though really proportional to their sines.

$$\begin{aligned} C &= 90°, & c &= 18.75, \\ B &= 55° \, 32', & b &= ?, \\ A &= 34° \, 28', & a &= ?. \end{aligned}$$

Since

$$C > B > A.$$

The decimal point in b and a must be placed so that $c>b>a$. This gives $a=15.5$, $b=10.6$.

Given an Acute Angle and the Opposite Side

(Wentworth and Smith Trigonometry, page 64)

Given

$$A = 62° \, 10', \quad a = 78.$$

The logarithmic solution gives

$$b = 41.18, \quad c = 88.20.$$

Check:

$$\frac{A}{S} \left| \frac{a}{\sin A} \right| \frac{b}{\sin B} \left| \frac{c}{\sin C} \right.$$

Substituting:

$$\frac{A}{S} \left| \frac{\textit{Opposite } 78}{\textit{Set } 62° \, 10'} \right| \frac{\textit{Read } 412}{\textit{Opposite } 27° \, 50'} \left| \frac{\textit{Read } 882}{\textit{Opposite } 90°} \right.$$

The decimal point:

$$\begin{aligned} C &= 90°, & c &= ?, \\ A &= 62° \, 10', & a &= 78.0, \\ B &= 27° \, 50', & b &= ?. \end{aligned}$$

Since $C>A>B$ the decimal point in c and b must be placed so that $c>a>b$ which gives $c=88.2$, $b=41.2$.

Given an Acute Angle and the Adjacent Side

(Wentworth and Smith Trigonometry, page 64)

Given

$$A = 50° \, 2', \quad b = 88.$$

The logarithmic solution gives

$$a = 105, \quad c = 137,$$

Since

$$A = 50° 2', \quad B = 39° 58'$$

Check:

$$\frac{A}{S} \left| \frac{b}{\sin B} \right| \frac{a}{\sin A} \left| \frac{c}{\sin C} \right.$$

Substituting:

$$\frac{A}{S} \left| \frac{\textit{Opposite } 88}{\textit{Set } 39° 58'} \right| \frac{\textit{Read } 102}{\textit{Opposite } 50° 2'} \left| \frac{\textit{Read } 137}{\textit{Opposite } 90°} \right.$$

In this case 50° 2′ projects beyond the right end of the rule. Set the indicator opposite the left index of the slide and then move the slide, setting the right index opposite the indicator. Now opposite 50° 2′ on S read 102 on A and opposite 90° on S read 137 on A.

The decimal point:

$$\begin{aligned} C &= 90°, & c &= ?, \\ A &= 50° 2', & a &= ?, \\ B &= 39° 58', & b &= 88. \end{aligned}$$

Since $C > A > B$, the decimal point in c and a must be placed so that $c > a > b$ which gives $c = 137$, $a = 102$.

Given the Hypotenuse and a Side

(Wentworth and Smith Trigonometry, page 65)

Given

$$a = 47.55, \quad c = 58.4$$

The logarithmic solution gives

$$A = 54° 31', \quad B = 35° 29', \quad b = 33.90$$

Check:

$$\frac{A}{S} \left| \frac{c}{\sin C} \right| \frac{a}{\sin A} \left| \frac{b}{\sin B} \right.$$

Substituting:

$$\frac{A}{S} \left| \frac{\textit{Opposite } 58.4}{\textit{Set } 90°} \right| \frac{\textit{Opposite } 476}{\textit{Read } 54° 31'} \left| \frac{\textit{Read } 339}{\textit{Opposite } 35° 29'} \right.$$

The decimal point:

$$\begin{aligned} C &= 90°, & c &= 58.4, \\ A &= 54° 31', & a &= 47.6, \\ B &= 35° 29', & b &= ?. \end{aligned}$$

Since $C > A > B$, the decimal point in b must be placed so that $c > a > b$ which gives $b = 33.9$.

Given the Two Sides

(Wentworth and Smith Trigonometry, page 65)

Given:

$$a = 40, \quad b = 27.$$

The logarithmic solution gives

$$A = 55° 59', \quad B = 34° 1', \quad c = 48.26.$$

Check:

$$\frac{A}{S} \left| \frac{a}{\sin A} \right| \frac{b}{\sin B} \left| \frac{c}{\sin C} \right.$$

Substituting:

$$\frac{A}{S} \left| \frac{\textit{Opposite } 40}{\textit{Set } 55° 59'} \right| \frac{\textit{Opposite } 27}{\textit{Read } 34° 1'} \left| \frac{\textit{Read } 483}{\textit{Opposite } 90°} \right.$$

The decimal point:

$$\begin{aligned} C &= 90°, & c &= ?, \\ A &= 55° 59', & a &= 40, \\ B &= 34° 1', & b &= 27. \end{aligned}$$

Since $C > A > B$ the decimal point in c must be placed so that $c > a > b$ which gives $c = 48.3$.

Oblique Triangles

Given Two Angles and a Side

(Wentworth and Smith Trigonometry, page 110)

Given

$$a = 24.31, \quad A = 45° 18', \quad \text{and} \quad B = 22° 11'$$

The logarithmic solution gives

$$b = 12.91, \quad c = 31.59, \quad C = 112° 31'$$

Check:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

Substituting:

$$\frac{A}{S} \left| \frac{\textit{Opposite } 24.31}{\textit{Set } 45° 18'} \right| \frac{\textit{Read } 129}{\textit{Opposite } 22° 11'} \left| \frac{\textit{Read } 316}{\textit{Opposite } 67° 29'} \right.$$

(*Since* sin *112° 31'*

= sin *67° 29'*

The decimal point:

$$\begin{aligned} C &= 112^\circ\,31', & c &= ?, \\ A &= 45^\circ\,18', & a &= 24.31, \\ B &= 22^\circ\,11', & b &= ?. \end{aligned}$$

Since $C > A > B$, the decimal point in b and c must be placed so that $c > a > b$, which gives $b = 12.9$, $c = 31.6$.

Given Two Sides and the Angle Opposite One of these Sides.

(Wentworth and Smith Trigonometry, p. 114, Ex. 9)

Given:

$$a = 177.01, \quad b = 216.45, \quad A = 35^\circ\,36'\,20''$$

The first logarithmic solution gives:

$$B = 45^\circ\,23'\,28'', \quad c = 300.29, \quad C = 99^\circ\,0'\,12''.$$

Check:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Substituting:

$\frac{A}{S}$			
A	*Opposite 177 (on left half)*	*Opposite 216 (left half)*	*Read 300*
S	*Set 35° 36′*	*Read 45° 23′*	*Opposite 81 (180°–99°)*

The decimal point: Arranging in order of magnitude

$$\begin{aligned} C &= 99^\circ\,0'\,12'', & c &= ?, \\ B &= 45^\circ\,23'\,28'', & b &= 216, \\ A &= 35^\circ\,36'\,20'', & a &= 177. \end{aligned}$$

Since $C > B > A$, the decimal point in c must be placed so that $c > b > a$ which gives $c = 300$.

The second solution gives

$$\begin{aligned} B &= 134^\circ\,36'\,32'', \\ C &= 9^\circ\,47'\,8'', \\ c &= 51.68. \end{aligned}$$

Check:

A	*Opposite 177 (on left half)*	*Opposite 216 (left half)*	*Read 517*
S	*Set 35° 36′*	*Read 45° 23′ or 180°–45° 23′ = 134° 37′*	*Opposite 9° 47′ (Reverse the slide)*

The decimal point:

$$\begin{aligned} B &= 134^\circ\,37', & b &= 216, \\ A &= 35^\circ\,36', & a &= 177, \\ C &= 9^\circ\,47', & c &= ?. \end{aligned}$$

Since $B>A>C$, the decimal point in c must be placed so that $b>a>c$, which gives $c=51.7$.

Given Two Sides and the Included Angle.

(Wentworth and Smith Trigonometry, page 121)

Given:

$$C = 63^\circ\,35'\,30'', \quad a = 748, \quad b = 375.$$

The logarithmic solution gives:

$$A = 86^\circ\,23'\,9'', \quad B = 30^\circ\,1'\,21'', \quad c = 671.27.$$

Check:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$\frac{A}{S}$	*Opposite 748 (left half)* / *Set 86° 23′*	*Opposite 375* / *Read 30° 1′*	*Read 671* / *Opposite 63° 35′*

The decimal point:

$$\begin{aligned} A &= 86^\circ, & a &= 748, \\ C &= 63^\circ, & c &= ?, \\ B &= 30^\circ, & b &= 375. \end{aligned}$$

Since $A>C>B$, the decimal point in c must be placed so that $a>c>b$ which gives 671.

Given Three Sides

(Wentworth and Smith, Trigonometry, page 126, Ex.1)

Given:

$$a = 3.41, \quad b = 2.60, \quad c = 1.58.$$

The logarithmic solution gives

$$A = 106^\circ\,46'\,40'', \quad B = 46^\circ\,53'\,14'', \quad C = 26^\circ\,20'\,6''.$$

Check: If C has been found by subtracting the sum of A and B from 180°, use the sine formula

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

A	*Opposite 3.41*	*Opposite 2.6*	*Opposite 1.58*
S	*Set 73° 13′*	*Read 46° 53′*	*Read 26° 20′*
	(180°–106° 46′)		

If C has been found by logarithms, the best check is

$$A + B + C = 180^\circ.$$

The results of the persistent use of the slide rule as a check are increased accuracy and speed. If an error has been made, it is instantly detected by the slide rule, while the usual check by logarithms or natural functions takes considerable time and is sufficiently tedious to prevent many students from undertaking it at all.

Again, the slide rule often locates the error in a particular portion of the work. In the first case of oblique triangles, if one unknown is correct and the other incorrect, the slide rule notes that fact instantly.

An indirect result of using the slide rule for a check in trigonometry is that the student learns to perform ordinary calculations such as occur in science and business with speed and a minimum of mental effort. From an educational viewpoint the small amount of time spent in teaching the slide rule is well worth while on the side of thrift in saving time and increased accuracy in calculation.

Notice to Subscribers

The *Official Ballot* for the election of officers of the *National Council of Teachers of Mathematics* will be found on page 62 of this issue. Every member of the Council should vote. If you do not wish to cut out and mark the ballot on page 62 make a copy on a 3 x 5 card and send it to the secretary, Mr. Schreiber.

Relating Art and Mathematics

By Bertha Luella Weir
Glen Rock, New Jersey

A very important problem in junior high school mathematics is to find concrete materials for the cultural values of mathematics, such as the appreciation of its beauty and to constructively develop the imagination of the pupil. Marie Gugle's article on Dynamic Symmetry in "The Third Yearbook of the National Council" inspired me to introduce the root-two rectangle in connection with the work in square root and the Pythagorean Theorem.

That the imagination of the pupils had been stimulated was evident when one saw the wealth of designs which they made. The making of a pleasing design in the rectangle was the first step in our study of this very fascinating topic. The boys and girls decided that the ultimate goal of a design was that it be one that could be applied to some object to enhance its beauty. Root-two acquired a new and more lovely meaning. It decorated pillow tops, resplendent in subdued tones of aurora work and spread to the art classes. Tea-tiles, blotter corners and book ends were popular expressions of the gracefulness of Dynamic Symmetry.

The children reveled in these designs and a fine sense of the appreciation of the work of their classmates was developed. A boy whose financial circumstances did not permit the application of his designs made a very beautiful design for a pillow top. The class decided that it was the best one that had been made. Another boy borrowed the design, applied it to a pillow top and it was presented to the teacher by the two boys.

The designs brought the home and school closer together as they soon became a topic of home conversation. The parents were so interested in them that the class decided we should have an exhibit in order that their families might see them.

The work certainly developed a fine sense of beauty in the pupils. One of the boys said he had disliked modernistic designs but he liked the modernistic ones that were made from Dynamic Symmetry. The boys and girls admired the very pleasing proportions that result in the use of Dynamic Symmetry.

We Pay Tribute to Archimedes

THE BUST of Archimedes which appears on the front outside cover of this issue of the MATHEMATICS TEACHER is to be a permanent affair. He has been chosen not only because he is one of the ancient mathematicians, but also because he was one of the greatest ones. In fact we are surer of many of his accomplishments than we are of many other famous mathematicians. Professor Smith* speaks of him as follows:

> Leibniz praised his genius by saying that those who knew his works and those of Apolonius marvelled less at the discoveries of the greatest modern scholars. These words are justified, for Archimedes anticipated nearly two thousand years some of the ideas of Newton and his contemporaries, and in the application of mathematics to mechanics he had no equal in ancient times. One of the Italian historians of mathematics uses the happy phrase that he had "a genius more divine than human," and Pliny calls him "the God of mathematics," a phrase which one of his French translators felicitously renders as "the Homer of geometry."

Archimedes (287-212 B.C.) studied in the university at Alexandria and lived in Sicily. He loved science so much that he held it undesirable to apply his information to practical use. But so great was his mechanical ability that when a difficulty had to be overcome the government often called on him. He introduced many inventions into the everyday lives of the people.

His life is exceedingly interesting. Read the stories of his detection of the dishonest goldsmith; of the use of burning-glasses to de-

* Smith, David Eugene, *History of Mathematics*. Vol. I, page 111, Ginn and Co. 1923.

stroy the ships of the attacking Roman squadron; of his clever use of a lever device for helping out Hiero, who had built a ship so large that he could not launch it off the slips; of his screw for pumping water out of ships and for irrigating the Nile valley. He devised the catapults which held the Roman attack for three years. These were so constructed that the range was either long or short and so that they could be discharged through a small loophole without exposing the men to the fire of the enemy.

When the Romans finally captured the city Archimedes was killed, though contrary to the orders of Marcellus, the general in charge of the siege. It is said that soldiers entered Archimedes' study while he was studying a geometrical figure which he had drawn in sand on the floor. Archimedes told a soldier to get off the diagram and not to spoil it. The soldier, insulted at having orders given to him and not knowing the old man, killed him.

The Romans erected a splendid tomb with the figure of a sphere engraved on it. Archimedes had requested this to commemorate his discovery of the two formulas: the volume of a sphere equals two-thirds that of the circumscribing right cylinder, and the surface of a sphere equals four times the area of a great circle. You may also read an interesting account by Cicero of his successful efforts to find Archimedes' tomb.

OFFICIAL BALLOT

For the Election of Officers at the February 21, 1930, Meeting of the

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

For President 1930-32
Vote for One

☐ Everett, John P.
Kalamazoo, Michigan

☐ Taylor, E. H.
Charleston, Illinois

For Second Vice-President, 1930-32
Vote for One

☐ Schlauch, W. S.
New York, New York

☐ Stone, John C.
Montclair, New Jersey

For Members of Board of Directors, 1930-33
Vote for Three

☐ Austin, C. M.
Oak Park, Illinois

☐ Barber, H. C.
Exeter, New Hampshire

☐ Betz, William
Rochester, New York

☐ Hildebrandt, Martha
Maywood, Illinois

☐ Miller, Florence Brooks
Cleveland, Ohio

☐ Stokes, C. N.
Minneapolis, Minnesota

Please mark this Ballot at once and mail same to Edwin W. Schreiber, Secretary, 434 W. Adams St., Macomb, Illinois.

Program

ANNUAL MEETING OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

CHALFONTE-HADDON HALL

ATLANTIC CITY

February 21 and 22, 1930

(Chairman of Local Committee, B. W. LIDELL, Atlantic City High School)

10:00 Friday Morning, February 21

Joint Meeting of the Directors and the Local Committee
11:30. Directors' Luncheon

OPEN MEETINGS

2:30 Friday Afternoon, February 21

ANNUAL BUSINESS MEETING

7:45 Friday Evening, February 21

Address of Welcome............................Henry P. Miller
Atlantic City High School

Greetings from Local Mathematicians.............Fletcher Durrell
Belleplain, N.J.

Response for the Council.........First Vice-President C. M. Austin
Oak Park High School, Oak Park, Ill.

A Year of Plane and Solid Geometry................John C. Stone
New Jersey State Teachers College, Montclair, N.J.

A Modern View of Geometry.....................Oswald Veblin
Princeton University, Princeton, N.J.

9:00 Saturday Morning, February 22

The Fifth Year Book: The Teaching of Geometry.......Wm. Betz
Specialist in Mathematics, Rochester, N.Y.

Rebuilding Geometry..........................George W. Evans
Boston, Mass.

Geometry Measures Land........................W. R. Ransom
Tufts College, Medford, Mass.

Report of the Joint Committee on Geometry—....Dunham Jackson (Chairman), University of Minnesota

Discussion..............................Led by Hallie S. Poole Buffalo, N.Y.

12:00 Saturday, February 22

DIRECTORS' LUNCHEON

2:00 Saturday Afternoon, February 22

Geometry in Junion High School...................Marie Gugle
Assistant Superintendent of Schools, Columbus, Ohio

The Introduction to Plane Geometry................E. H. Taylor
State Teachers College, Charleston, Ill.

From Plane to Solid...........................E. W. Schreiber
Western Illinois State Teachers College, Macomb, Ill.

A Professionalized Course in the Teaching of Geometry.......
..........H. C. Christofferson, Miami University, Oxford, Ohio

Discussion..............................Led by C. M. Austin

6:00 Saturday Evening, February 22

Greetings from Guests of Honor

Geometry as Preparation for College...............W. R. Longley
Yale University

This is a strong program. An interesting and profitable time is assured. Come to Atlantic City and take part in these beginnings of curriculum revision in geometry. Please spread the notice of this meeting as widely as possible—by word of mouth and by letter. Also help to get new members. Send $2.00 to Editor W. D. Reeve, 525 West 120th Street, New York City.

Rooms in Atlantic City are not high priced but are hard to get. Reservations should be made at once.

HARRY C. BARBER, *President*

NEWS NOTES

The Mathematics Club of Cleveland, Ohio, has appointed a committee to investigate the feasibility of interscholastic competition in mathematics. Any reports on that subject by anyone who has had experience will be gratefully received.

If the investigation bears any fruit that is worth while it will be submitted to the Mathematics Teacher for publication.

P. Stroup, *Chairman*
West High School
Cleveland, Ohio

The Tenth Annual Ohio State Educational Conference

"Reaching the Individual" will be the keynote of the Tenth Annual Ohio State Educational Conference to be held in Columbus, April 3, 4, 5, 1930. Mr. Robert M. Hutchins, president of the University of Chicago, will speak at the Thursday night general session. Mr. E. H. Sothern, well-known actor and dramatic reader, will give a series of readings from Shakespeare Friday night.

Sections 7 and 19 (Mathematics) of the New York Society for the Experimental Study of Education held a joint dinner meeting on Saturday, November 23, at the Men's Faculty Club of Columbia University. Professor David Eugene Smith spoke on "Revolution in Mathematics." Brief speeches appropriate for the occasion were made by Mr. H. H. Wright, District Superintendent, High School Division, New York City, and Miss Sheerin, Chairman of Section 7. Three hundred were present at the meeting.

The Detroit Mathematics Club has held two meetings this fall. The speaker at the first meeting on October 3 was D. B. P. Fowler, Headmaster of the Tower Hill School of Wilmington, Delaware. At the second meeting on December 5, Dr. J. P. Everett of the Western State Teachers College of Kalamazoo, Michigan, spoke on "The Fundamental Skills of Algebra."

The total number of members in the the club's history.

Six schools in Detroit now have 100% membership in the National Council of Teachers of Mathematics. They are the Sherrard, the Nolan, the Cleveland, the Condon, the Jefferson, and the McKenzie, all intermediate club last year was 280—the largest in schools.

The following letter to the Editor of the Mathematics Teacher is the kind of thing that encourages us in the work that we are trying to do for the National Council:

"Dear Editor:—Enclosed you will find a check for $2.00 as a renewal of my subscription to The Mathematics Teacher beginning with the January 1930 issue.

"I surely do enjoy this magazine. Every worthwhile teacher of mathematics should be a subscriber to it. I became one only by accident. Never was it mentioned to me by my university teachers of mathematics. If every Professor of the teaching of mathematics would make his students acquainted with the merits of this magazine I feel sure that most of them would subscribe for it as soon as they became actual teachers of mathematics in Junior or Senior high schools or colleges."

Kathryn E. Baum